Projectile Thrust-Drag Optimization with External Burning

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Theme

AN analysis using the method of characteristics with heat addition and the Crocco-Lees base flow model is described for calculating the external supersonic flow with heat addition about the wake of a 5-in. projectile. Various amounts and locations of heat addition are found to generate a net thrust, and an optimum location for the combustion zone is predicted. Good agreement is observed between calculated and experimental base pressure measurements using a free-jet combustion simulator. The results from computing base pressure ratio over a wide range of Reynolds numbers are also presented.

Contents

The method for determining the base-pressure ratio of a projectile with external burning centers around the model used to describe the complex base flow region. Following the two-dimensional planar study of Strahle,[1] the model used in this axisymmetric analysis[2] was the Crocco-Lees[3] base-flow theory, which considers the base flow as a separated boundary layer merging into a free-jet profile and derives integral conservation equations to describe the mean values of the flow quantities. From these integrals, a single ordinary differential equation is derived that has a saddle-point singularity and is the same for both axisymmetric and two-dimensional planar flow. This equation uses an empirical shear-layer mixing coefficient k, which was transitioned from its laminar value k_L to its turbulent value k_T during the calculation process by means of an integrated normal distribution. Two transition Reynolds numbers were used to identify the start and completion of the boundary-layer transition, either at the projectile base or along the shear layer.

An iterative method was developed to integrate the ordinary differential equation along the base-flow shear layer and straddle the saddle-point singularity. Repeated application of this straddle method determines the base pressure ratio within closer limits above and below the correct value and thus permits calculation to the required accuracy. This method used the method of characteristics with heat addition to compute the supersonic flowfield outside the base-flow shear layer and matched the shear-layer velocity with that of the intersecting-right-running characteristic line at each step of the integration process. To achieve the straddle condition, the assumed value of base-pressure ratio p_b/p_∞ was systematically increased after each integration until the shear-layer integration went above the saddle-point singularity or critical point. The interval between these last two base-pressure-ratio points that straddle the critical point was divided into ten parts, and by systematically stepping through these new values the next straddle condition was achieved. The base corner expansion or compression fan was calculated for each new assumed value of base-pressure ratio before starting the new shear-layer integration.

To carry out the experimental investigation, several axisymmetric free-jet nozzles were cast of epoxy by pouring about a wax plug mold. The wax plugs were formed by turning against different aluminum templates, and the different nozzle-wall contours represented various amounts and locations of external burning. Since the nozzle walls generated compression waves that impinged on the base-flow region, the nozzle duplicated the compression waves that would be generated by burning in the freestream. Each nozzle was tested with a coaxial center body, measuring base pressure and using Schlieren photography. The nozzle-throat position, relative to the base, was adjusted with spacers to establish the effect of heat-zone location in the axial z direction.

Some experimental and calculated values of p_b/p_∞ are compared in Fig. 1. Good agreement was obtained for flow with and without heat addition. For a given amount of heat addition, it was found experimentally that a loss of p_b/p_∞ would occur when the compression waves impinged ahead of the base, $z/r<0$. This was attributed to adverse pressure-gradient effects on the boundary layer approaching the projectile base, which resulted in an increase of the shear-layer mixing coefficient k. This effect was not considered in the computer calculation. It was also found that compression waves that impinge on the shear layer far downstream of the critical point do not affect the base-pressure ratio since the mixing region jet flow has accelerated to supersonic condition. However, compression wave impingement on the

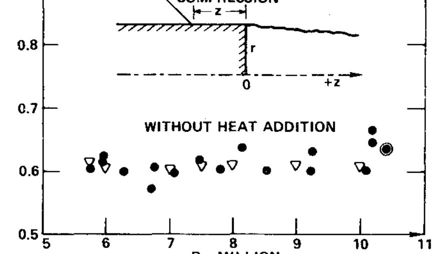

Fig. 1 Comparison of experimental and computed values of base pressure ratio vs Reynolds number at Mach 2.0 with and without simulated heat addition.

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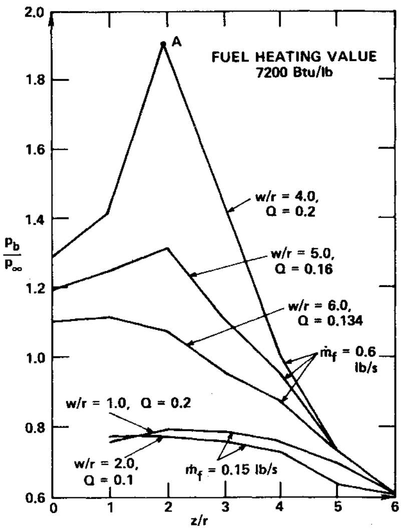

Fig. 2 Computed values of base pressure ratio at Mach number of 2.0 with external burning.

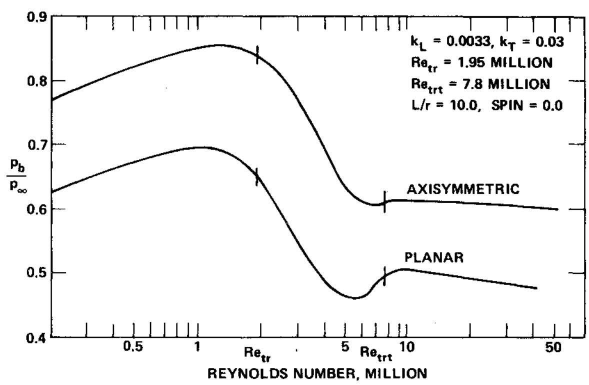

Fig. 3 Computed values of p_b/p_∞ vs Reynolds number, Mach 2.0, no heat addition.

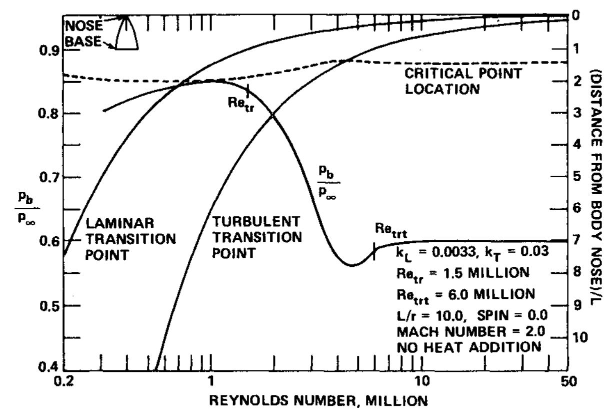

Fig. 4 Computed values of p_b/p_∞, transition points and critical point.

shear layer that continued slightly downstream of the critical point was found to be desirable.

To provide some estimate of the optimum location for external heat addition, a fixed fuel flow was assumed to be burned in zones of different length and location. The heat zones initial inner streamline radius was $2.0r$, and $3.0r$ was used for the outer streamline radius, where r is the radius of the projectile base. The length of the heat zone w/r was assigned integer values from 1 through 6. The heat zone with $w/r = 1.0$ was assigned a value of heat addition per unit mass of $q = 0.2$. This value of q was decreased by $1/(w/r)$ for the other five zones so that the same total heat would be added to each flowfield. The heat zones were positioned along the flow axis starting with compression at the base and moving downstream in integer values of radius. The results of two of these computations are shown as the lower curves in Fig. 2. The gain in p_b/p_∞ for $m = 0.15$ lb/sec is seen to be fairly insensitive to heat-zone size and placement.

Another set of similar calculations was made based on $q = 0.8$ at $w/r = 1.0$. This value of q corresponded to a fuel flow of 0.6 lb/sec at 23,000 ft altitude. The first three values of w/r, 1.0, 2.0, and 3.0 would not compute because of thermal choking where the external flow was driven to a sonic condition. With the last three values of w/r, 4.0, 5.0, and 6.0, the external flow remained supersonic, and the results are shown as the upper three curves in Fig. 2. For the $w/r = 4.0$ curve, the high value of $p_b/p_\infty = 1.91$ at A represents a sixfold increase in p_b/p_∞ over the two bottom curves in Fig. 2, with a fourfold increase in fuel flow, and corresponds to a specific impulse of over 230. The heat addition for this optimum location was started at $z/r = -1.2$, and the initial compression on the shear layer occurred at $z/r = +2.0$.

The procedure used to transition the shear-layer mixing coefficient from its laminar to its turbulent value permitted base-pressure-ratio calculation over a wide range of Reynolds number as shown in Fig. 3. The laminar transition Reynolds number Re_{tr} and the turbulent transition Reynolds number Re_{trt} were used to establish the normal distribution's 2% beginning and 98% end of mixing-coefficient transition. The curves in Fig. 3 were found to be in good agreement with experiment.[4] Re_{tr} is where the laminar boundary layer at the base starts transition to turbulent, and Re_{trt} is where the base boundary layer is completely turbulent, $Re_{trt} \doteq 4\ Re_{tr}$. At low base Reynolds number Re_b and completely laminar flow, it was necessary to increase the laminar mixing in direct proportion to the increase of the base boundary-layer thickness, $k_L' = k_L\ (Re_{tr}/Re_b)^{1/2}$. The laminar and turbulent transition points and the critical point locations versus Reynolds number were calculated and plotted in Fig. 4 with p_b/p_∞ to show their interrelation.

Conclusion

With the method of characteristics and the Crocco-Lees base flow model, one can accurately predict the base-pressure ratio of a two-dimensional or axisymmetric body in supersonic flow with moderate external burning for various Mach and Reynolds numbers. Repeated application of the calculation procedure permits optimization of the heat addition zone for maximum base-pressure ratio.

References

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